Supplemental Appendix:
Navigating the 'Problem from Hell': A Guide to Climate Damages

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Online Appendix

A Analysis for Section 2

Consider a world with two regions, denoted A and B, each comprised of small actors who take the other region's choices as given. In period t, region j produces quantity $Y_t^j \triangleq Y^j(\ell_{Yt}^j, K_t^j, w_t^j)$ of a nonstorable good using labor ℓ_{Yt}^j, capital K_t^j, and weather inputs w_t^j. Region j consumes quantities q_{At}^j and q_{Bt}^j of the goods produced by regions A and B, respectively. Its representative agent derives welfare $W_t^j \triangleq W^j(q_{At}^j, q_{Bt}^j)$ from that consumption bundle, with W^j increasing in each argument and concave. Using the price of region A's good as the numeraire, region A's time t budget constraint is $Y_t^A = q_{At}^A + p_t q_{Bt}^A$ and region B's time t budget constraint is $p_t Y_t^B = q_{At}^B + p_t q_{Bt}^B$. The market for the good produced by region j clears with $Y_t^j = q_{jt}^A + q_{jt}^B$. Each representative agent takes the terms of trade and the other region's capital stock as given, so that regions do not act strategically.

Region j is endowed with L^j units of labor. It allocates ℓ_{Yt}^j units of that labor to producing its good, and it allocates ℓ_{Kt}^j units of that labor to producing capital. The labor market clears with $\ell_{Yt}^j + \ell_{Kt}^j = L^j$. Region j's capital stock K_t^j depreciates at per-period rate $\delta \in (0,1)$ and increases as $f(\ell_{Kt}^j)$, with $f, f' > 0$ and $f'' < 0$. Capital and labor are here immobile across regions. All agents observe capital and weather in all regions.

The representative agent in region A at time t, with discount rate $r > 0$, solves the following

*ⓡ indicates that author order was randomized.

infinite-horizon problem, taking the path of p as given:

$$\begin{aligned}\max_{\ell^A_{Ys}(\cdot),\ell^A_{Ks}(\cdot),q^A_{As}(\cdot),q^A_{Bs}(\cdot)} \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} E_t\left[W^A(q^A_{As}, q^A_{Bs})\right] \\ \text{s.t. } K^A_{s+1} =& f(\ell^A_{Ks}) + (1-\delta)K^A_s \\ Y^A(\ell^A_{Ys}, K^A_s, w^A_s) =& q^A_{As} + p_s q^A_{Bs} \\ L^A =& \ell^A_{Ys} + \ell^A_{Ks},\end{aligned}$$

where the policies $\ell^A_{Ys}(\cdot)$, $\ell^A_{Ks}(\cdot)$, $q^A_{As}(\cdot)$, and $q^A_{Bs}(\cdot)$ are functions of the time s state vector (comprising K^j_s, K^k_s, w^j_s, and w^k_s), and where E_t indicates expectations at the common time t information set. The representative agent in region B at time t solves an analogous problem, with the budget constraint adjusted appropriately. Optimal input and consumption choices in region j solve the following Bellman equation:

$$V^j(K^j_t, K^k_t, w^j_t, w^k_t) = \max_{\ell^j_{Yt},\ell^j_{Kt},q^j_{At},q^j_{Bt}} \left\{ W^j(q^j_{At}, q^j_{Bt}) + \frac{1}{1+r} E_t\left[V^j(K^j_{t+1}, K^k_{t+1}, w^j_{t+1}, w^k_{t+1})\right] \right\}, \tag{A-1}$$

with k indexing the other region. Define $V^j_t \triangleq V^j(K^j_t, K^k_t, w^j_t, w^k_t)$.

Substituting in the budget constraint and the transition for capital in region A, and then substituting in region A's labor constraint, equation (A-1) for region A is equivalent to:

$$\begin{aligned}V^A(K^A_t, K^B_t, w^A_t, w^B_t) = \max_{\ell^A_{Yt},q^A_{Bt}} \Big\{ & W^A(Y^A(\ell^A_{Yt}, K^A_t, w^A_t) - p_t q^A_{Bt}, q^A_{Bt}) \\ & + \frac{1}{1+r} E_t\left[V^A\Big(f(L^A - \ell^A_{Yt}) + (1-\delta)K^A_t, K^B_{t+1}, w^A_{t+1}, w^B_{t+1}\Big)\right]\Big\}.\end{aligned} \tag{A-2}$$

The first-order condition for q^A_{Bt} yields:

$$p_t = \frac{\frac{\partial W^A(q^A_{At}, q^A_{Bt})}{\partial q^A_{Bt}}}{\frac{\partial W^A(q^A_{At}, q^A_{Bt})}{\partial q^A_{At}}}. \tag{A-3}$$

Substituting for q^A_{At} from the budget constraint, this equation implicitly defines q^A_{Bt} as a function of p_t and Y^A_t, and the analogous equation for region B implicitly defines q^B_{At} as a function of p_t and Y^B_t, from which the budget constraint yields q^B_{Bt}. Market-clearing for region B's production good becomes $Y^B_t = q^A_{Bt}(p_t, Y^A_t) + q^B_{Bt}(p_t, Y^B_t)$, which implicitly defines p_t as $p(Y^A_t, Y^B_t)$.

Because regions take the terms of trade as given, problem (A-2) becomes:

$$\begin{aligned} &V^A(K_t^A, K_t^B, w_t^A, w_t^B) \\ &= \max_{\ell_{Yt}^A} \Bigg\{ W^A\Big(Y^A(\ell_{Yt}^A, K_t^A, w_t^A) - p_t q_B^A(p_t, Y^A(\ell_{Yt}^A, K_t^A, w_t^A)),\ q_B^A(p_t, Y^A(\ell_{Yt}^A, K_t^A, w_t^A))\Big) \\ &\qquad + \frac{1}{1+r} E_t\left[V^A\Big(f(L^A - \ell_{Yt}^A) + (1-\delta)K_t^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B\Big)\right]\Bigg\}. \end{aligned} \tag{A-4}$$

The first-order condition for ℓ_{Yt}^A is

$$0 = \left[\frac{\partial W_t^A}{\partial q_{At}^A} + \frac{\partial q_{Bt}^A}{\partial Y_t^A}\left(\frac{\partial W_t^A}{\partial q_{Bt}^A} - p_t \frac{\partial W_t^A}{\partial q_{At}^A}\right)\right]\frac{\partial Y_t^A}{\partial \ell_{Yt}^A} - f'(\ell_{Kt}^A)\frac{1}{1+r}E_t\left[\frac{\partial V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial K_{t+1}^A}\right].$$

Substitute p_t from (A-3):

$$0 = \frac{\partial W_t^A}{\partial q_{At}^A}\frac{\partial Y_t^A}{\partial \ell_{Yt}^A} - f'(\ell_{Kt}^A)\frac{1}{1+r}E_t\left[\frac{\partial V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial K_{t+1}^A}\right]. \tag{A-5}$$

The optimal choice of ℓ_{Yt}^A equates the marginal benefit in terms of today's production and the marginal cost in terms of forgone capital in the future.

Equation (A-5) implicitly defines ℓ_{Yt}^A as a function of ℓ_{Yt}^B, K_t^A, K_t^B, $\tilde{w}_t^A$, $\tilde{w}_t^B$, and C. The analogous equation for region B implicitly defines ℓ_{Yt}^B as a function of ℓ_{Yt}^A, K_t^A, K_t^B, $\tilde{w}_t^A$, $\tilde{w}_t^B$, and C. Recalling that $\tilde{w}_t^A$ and $\tilde{w}_t^B$ (mean-zero random variables reflecting stochasticity in weather) are independent of C,

$$\frac{\mathrm{d}\ell_{Yt}^A}{\mathrm{d}C} = \frac{\partial \ell_{Yt}^A}{\partial C} + \frac{\partial \ell_{Yt}^A}{\partial K_t^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C} + \frac{\partial \ell_{Yt}^A}{\partial K_t^B}\frac{\mathrm{d}K_t^B}{\mathrm{d}C} + \frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}\frac{\mathrm{d}\ell_{Yt}^B}{\mathrm{d}C}. \tag{A-6}$$

Substitute the analogous equation for region B and collect terms with $\mathrm{d}\ell_{Yt}^A/\mathrm{d}C$ on the left-hand side:

$$\begin{aligned} \left[1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A}\frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}\right]\frac{\mathrm{d}\ell_{Yt}^A}{\mathrm{d}C} &= \underbrace{\frac{\partial \ell_{Yt}^A}{\partial C} + \frac{\partial \ell_{Yt}^A}{\partial K_t^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C} + \frac{\partial \ell_{Yt}^A}{\partial K_t^B}\frac{\mathrm{d}K_t^B}{\mathrm{d}C}}_{\zeta_{At}} \\ &\quad + \frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}\Bigg[\underbrace{\frac{\partial \ell_{Yt}^B}{\partial C} + \frac{\partial \ell_{Yt}^B}{\partial K_t^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C} + \frac{\partial \ell_{Yt}^B}{\partial K_t^B}\frac{\mathrm{d}K_t^B}{\mathrm{d}C}}_{\zeta_{Bt}}\Bigg]. \end{aligned} \tag{A-7}$$

Applying the implicit function theorem to equation (A-5) and using the second-order condition for an optimum (which guarantees that the proportionality constant omitted below is positive), the term labeled ζ_{At} can be written as:

$$\zeta_{At} = \gamma_{At}^{now} + \gamma_{At}^{future} + \gamma_{At}^{past} + \gamma_{At}^{elsewhere},$$

where

$$\begin{aligned}
\gamma_{At}^{now} \propto & \frac{\partial W_t^A}{\partial q_{At}^A} \frac{\partial^2 Y_t^A}{\partial \ell_{Yt}^A \partial w_t^A} \theta^A + \left[\frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} + \frac{\partial q_{Bt}^A}{\partial Y_t^A} \left(\frac{\partial^2 W_t^A}{\partial q_{At}^A \partial q_{Bt}^A} - p_t \frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} \right) \right] \frac{\partial Y_t^A}{\partial \ell_{Yt}^A} \frac{\partial Y_t^A}{\partial w_t^A} \theta^A \\
& + \left[- \frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} \left(q_{Bt}^A + p_t \frac{\partial q_{Bt}^A}{\partial p_t} \right) + \frac{\partial^2 W_t^A}{\partial q_{At}^A \partial q_{Bt}^A} \frac{\partial q_{Bt}^A}{\partial p_t} \right] \frac{\partial Y_t^A}{\partial \ell_{Yt}^A} \frac{\partial p_t}{\partial Y_t^A} \frac{\partial Y_t^A}{\partial w_t^A} \theta^A, \\
\gamma_{At}^{future} \propto & - f'(\ell_{Kt}^A) \frac{1}{1+r} E_t \left[\frac{\partial^2 V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial K_{t+1}^A \partial w_{t+1}^A} \right] \theta^A, \\
\gamma_{At}^{past} \propto & \frac{\mathrm{d}K_t^A}{\mathrm{d}C} \Bigg\{ \frac{\partial W_t^A}{\partial q_{At}^A} \frac{\partial^2 Y_t^A}{\partial \ell_{Yt}^A \partial K_t^A} + \left[\frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} + \frac{\partial q_{Bt}^A}{\partial Y_t^A} \left(\frac{\partial^2 W_t^A}{\partial q_{At}^A \partial q_{Bt}^A} - p_t \frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} \right) \right] \frac{\partial Y_t^A}{\partial \ell_{Yt}^A} \frac{\partial Y_t^A}{\partial K_t^A} \\
& + \left[- \frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} \left(q_{Bt}^A + p_t \frac{\partial q_{Bt}^A}{\partial p_t} \right) + \frac{\partial^2 W_t^A}{\partial q_{At}^A \partial q_{Bt}^A} \frac{\partial q_{Bt}^A}{\partial p_t} \right] \frac{\partial Y_t^A}{\partial \ell_{Yt}^A} \frac{\partial p_t}{\partial Y_t^A} \frac{\partial Y_t^A}{\partial K_t^A} \\
& - (1-\delta) f'(\ell_{Kt}^A) \frac{1}{1+r} E_t \left[\frac{\partial^2 V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial [K_{t+1}^A]^2} \right] \Bigg\}, \\
\gamma_{At}^{elsewhere} \propto & \left[- \frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} \left(q_{Bt}^A + p_t \frac{\partial q_{Bt}^A}{\partial p_t} \right) + \frac{\partial^2 W_t^A}{\partial q_{At}^A \partial q_{Bt}^A} \frac{\partial q_{Bt}^A}{\partial p_t} \right] \frac{\partial Y_t^A}{\partial \ell_{Yt}^A} \frac{\partial p_t}{\partial Y_t^B} \frac{\partial Y_t^B}{\partial w_t^B} \theta^B \\
& - f'(\ell_{Kt}^A) \frac{1}{1+r} E_t \left[\frac{\partial^2 V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial K_{t+1}^A \partial w_{t+1}^B} \right] \theta^B \\
& + \left[- \frac{\partial^2 W_t^A}{\partial [q_{At}^A]^2} \left(q_{Bt}^A + p_t \frac{\partial q_{Bt}^A}{\partial p_t} \right) + \frac{\partial^2 W_t^A}{\partial q_{At}^A \partial q_{Bt}^A} \frac{\partial q_{Bt}^A}{\partial p_t} \right] \frac{\partial Y_t^A}{\partial \ell_{Yt}^A} \frac{\partial p_t}{\partial Y_t^B} \frac{\partial Y_t^B}{\partial K_t^B} \frac{\mathrm{d}K_t^B}{\mathrm{d}C} \\
& - (1-\delta) f'(\ell_{Kt}^A) \frac{1}{1+r} E_t \left[\frac{\partial^2 V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial K_{t+1}^A \partial K_{t+1}^B} \right] \frac{\mathrm{d}K_t^B}{\mathrm{d}C}.
\end{aligned}$$

ζ_{Bt} follows analogously. Substituting into equation (A-7), we find:

$$\begin{aligned}
\frac{\mathrm{d}\ell_{Yt}^A}{\mathrm{d}C} = & \frac{1}{1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A} \frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}} \left[\gamma_{At}^{now} + \gamma_{At}^{future} + \gamma_{At}^{past} + \gamma_{At}^{elsewhere} \right] \\
& + \frac{1}{1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A} \frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}} \frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B} \left[\gamma_{Bt}^{now} + \gamma_{Bt}^{future} + \gamma_{Bt}^{past} + \gamma_{Bt}^{elsewhere} \right]. \qquad \text{(A-8)}
\end{aligned}$$

Observe that:

$$\begin{aligned}\gamma_{Bt}^{elsewhere} \propto &\left[-\frac{\partial^2 W_t^B}{\partial q_{Bt}^B \partial q_{At}^B}\left(q_{Bt}^B - Y_t^B + p_t \frac{\partial q_{Bt}^B}{\partial p_t}\right) + \frac{\partial^2 W_t^B}{\partial [q_{Bt}^B]^2}\frac{\partial q_{Bt}^B}{\partial p_t}\right]\frac{\partial Y_t^B}{\partial \ell_{Yt}^B}\frac{\partial p_t}{\partial Y_t^A}\frac{\partial Y_t^A}{\partial w_t^A}\theta^A \\
&- f'(\ell_{Kt}^B)\frac{1}{1+r}E_t\left[\frac{\partial^2 V^B(K_{t+1}^B, K_{t+1}^A, w_{t+1}^B, w_{t+1}^A)}{\partial K_{t+1}^B \partial w_{t+1}^A}\right]\theta^A \\
&+ \left[-\frac{\partial^2 W_t^B}{\partial q_{Bt}^B \partial q_{At}^B}\left(q_{Bt}^B - Y_t^B + p_t \frac{\partial q_{Bt}^B}{\partial p_t}\right) + \frac{\partial^2 W_t^B}{\partial [q_{Bt}^B]^2}\frac{\partial q_{Bt}^B}{\partial p_t}\right]\frac{\partial Y_t^B}{\partial \ell_{Yt}^B}\frac{\partial p_t}{\partial Y_t^A}\frac{\partial Y_t^A}{\partial K_t^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C} \\
&- (1-\delta) f'(\ell_{Kt}^B)\frac{1}{1+r}E_t\left[\frac{\partial^2 V^B(K_{t+1}^B, K_{t+1}^A, w_{t+1}^B, w_{t+1}^A)}{\partial K_{t+1}^B \partial K_{t+1}^A}\right]\frac{\mathrm{d}K_t^A}{\mathrm{d}C}.\end{aligned}$$

Substituting for $\gamma_{Bt}^{elsewhere}$ in (A-8), using the implicit function theorem with equation (A-5) to obtain $\partial \ell_{Yt}^A / \partial \ell_{Yt}^B$, and collecting terms, we have:

$$\frac{\mathrm{d}\ell_{Yt}^A}{\mathrm{d}C} = \Gamma_{At}^{now} + \Gamma_{At}^{future} + \Gamma_{At}^{past} + \Gamma_{At}^{elsewhere}, \tag{A-9}$$

where:

$$\begin{aligned}\Gamma_{At}^{now} \triangleq &\frac{1}{1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A}\frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}}\Bigg\{\gamma_{At}^{now} \\
&+ \chi_{At}\left[-\frac{\partial^2 W_t^B}{\partial q_{Bt}^B \partial q_{At}^B}\left(q_{Bt}^B - Y_t^B + p_t \frac{\partial q_{Bt}^B}{\partial p_t}\right) + \frac{\partial^2 W_t^B}{\partial [q_{Bt}^B]^2}\frac{\partial q_{Bt}^B}{\partial p_t}\right]\frac{\partial Y_t^B}{\partial \ell_{Yt}^B}\frac{\partial p_t}{\partial Y_t^A}\frac{\partial Y_t^A}{\partial w_t^A}\theta^A\Bigg\}, \\
\Gamma_{At}^{future} \triangleq &\frac{1}{1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A}\frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}}\Bigg\{\gamma_{At}^{future} - \chi_{At} f'(\ell_{Kt}^B)\frac{1}{1+r}E_t\left[\frac{\partial^2 V^B(K_{t+1}^B, K_{t+1}^A, w_{t+1}^B, w_{t+1}^A)}{\partial K_{t+1}^B \partial w_{t+1}^A}\right]\theta^A\Bigg\}, \\
\Gamma_{At}^{past} \triangleq &\frac{1}{1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A}\frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}}\Bigg\{\gamma_{At}^{past} \\
&+ \chi_{At}\left[-\frac{\partial^2 W_t^B}{\partial q_{Bt}^B \partial q_{Bt}^A}\left(q_{Bt}^B - Y_t^B + p_t \frac{\partial q_{Bt}^B}{\partial p_t}\right) + \frac{\partial^2 W_t^B}{\partial [q_{Bt}^B]^2}\frac{\partial q_{Bt}^B}{\partial p_t}\right]\frac{\partial Y_t^B}{\partial \ell_{Yt}^B}\frac{\partial p_t}{\partial Y_t^A}\frac{\partial Y_t^A}{\partial K_t^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C} \\
&- \chi_{At}(1-\delta) f'(\ell_{Kt}^B)\frac{1}{1+r}E_t\left[\frac{\partial^2 V^B(K_{t+1}^B, K_{t+1}^A, w_{t+1}^B, w_{t+1}^A)}{\partial K_{t+1}^B \partial K_{t+1}^A}\right]\frac{\mathrm{d}K_t^A}{\mathrm{d}C}\Bigg\}, \\
\Gamma_{At}^{elsewhere} \triangleq &\frac{1}{1 - \frac{\partial \ell_{Yt}^B}{\partial \ell_{Yt}^A}\frac{\partial \ell_{Yt}^A}{\partial \ell_{Yt}^B}}\Bigg\{\gamma_{At}^{elsewhere} + \chi_{At}\left[\gamma_{Bt}^{now} + \gamma_{Bt}^{future} + \gamma_{Bt}^{past}\right]\Bigg\}, \\
\chi_{At} \propto &f'(\ell_{Kt}^B) f'(\ell_{Kt}^A)\frac{1}{1+r}E_t\left[\frac{\partial^2 V^A(K_{t+1}^A, K_{t+1}^B, w_{t+1}^A, w_{t+1}^B)}{\partial K_{t+1}^A \partial K_{t+1}^B}\right].\end{aligned}$$

The leading fraction in each of the four Γ terms is strictly positive in the regular case, in which investment in one region moves less than one for one with investment in the other. χ_{At} measures the spillover from region B's investment choice into region A's investment choice, operating through how region B's investment changes the subsequent prices p_{t+s} of its good. It scales the impact of climate-induced changes in region B's investment on region A's labor allocated to investment.

Now relate the channels within each Γ that are informally described in Section 2 to the formal expressions. Γ_{At}^{now} has four components. The first three are from γ_{At}^{now}: weather may directly alter the marginal product of labor (first term on first line in γ_{At}^{now}), weather may alter the value of labor's marginal product by directly affecting total output in region A (second term on first line in γ_{At}^{now}), and weather may alter prices (second line in γ_{At}^{now}). The fourth component reflects how weather may alter investment decisions in region B (terms with χ_{At} in Γ_{At}^{now}).

Γ_{At}^{future} has two components: how expected weather directly alters the marginal product of region A's capital in future periods (in γ_{At}^{future}), and how these expectations can affect region A's time t investment via region B's time t investment (term with χ_{At} in Γ_{At}^{future}).

In Γ_{At}^{past}, the time t capital stock may affect the time t marginal product of labor (first term on the first line in γ_{At}^{past}), the time t value of that marginal product at constant prices (second term on the first line in γ_{At}^{past}), time t prices (second line in γ_{At}^{past}), or the expected marginal product of capital in later periods, whether directly (third line in γ_{At}^{past}) or by affecting investment in region B (terms with χ_{At} in Γ_{At}^{past}). Let t_0 indicate some time prior to which C had been constant and prior to which there was no knowledge that C would ever change. Then, from the capital transition equation, labor market-clearing, and equation (A-9):

$$\frac{\mathrm{d}K_t^A}{\mathrm{d}C} = -\sum_{s=t_0}^{t-1}(1-\delta)^{t-1-s} f'(\ell_{Ks}^A)\left[\Gamma_{As}^{now} + \Gamma_{As}^{future} + \Gamma_{As}^{past} + \Gamma_{As}^{elsewhere}\right]. \tag{A-10}$$

The capital stock contains a memory of all past effects of climate change, which encompass how climate change affected weather in all earlier periods and how climate change affected expectations of subsequent weather held by agents in earlier periods.[1]

In $\Gamma_{At}^{elsewhere}$, current weather elsewhere on the planet affects prices in region A (first line in $\gamma_{At}^{elsewhere}$); anticipated weather elsewhere on the planet affects investment incentives in region A (second line in $\gamma_{At}^{elsewhere}$); past weather elsewhere around the planet affects capital stocks elsewhere around the planet and thereby affects both prices (third line in $\gamma_{At}^{elsewhere}$) and investment incentives (fourth line in $\gamma_{At}^{elsewhere}$) in region A; and all such weather can affect investment incentives in region B and thereby affect investment incentives in region A (terms with χ_{At} in $\Gamma_{At}^{elsewhere}$).

[1] The evaluation point for the partial derivatives in equation (4) in Section 3.2 can be different from the evaluation point for the partial derivatives in this appendix and in Section 2 because climate change alters the incoming capital stock. The evaluation points are the same only for the very first increment of surprising climate change.

To derive the effect of climate on period welfare in region A, observe from (A-4) that

$$\frac{\mathrm{d}W_t^A}{\mathrm{d}C} = \left[\frac{\partial W_t^A}{\partial q_{At}^A} + \frac{\partial q_{Bt}^A}{\partial Y_t^A}\left(\frac{\partial W_t^A}{\partial q_{Bt}^A} - p_t \frac{\partial W_t^A}{\partial q_{At}^A}\right)\right]\frac{\mathrm{d}Y_t^A}{\mathrm{d}C} + \left[-\frac{\partial W_t^A}{\partial q_{At}^A} q_{Bt}^A + \frac{\partial q_{Bt}^A}{\partial p_t}\left(\frac{\partial W_t^A}{\partial q_{Bt}^A} - p_t \frac{\partial W_t^A}{\partial q_{At}^A}\right)\right]\frac{\mathrm{d}p_t}{\mathrm{d}C}.$$

Using the equilibrium price from (A-3), we find:

$$\frac{\mathrm{d}W_t^A}{\mathrm{d}C} = \frac{\partial W_t^A}{\partial q_{At}^A}\left[\frac{\mathrm{d}Y_t^A}{\mathrm{d}C} - \underbrace{\left(\frac{\partial p_t}{\partial Y_t^A}\frac{\mathrm{d}Y_t^A}{\mathrm{d}C} + \frac{\partial p_t}{\partial Y_t^B}\frac{\mathrm{d}Y_t^B}{\mathrm{d}C}\right)}_{\mathrm{d}p_t/\,\mathrm{d}C} q_{Bt}^A\right].$$

We have the effect of climate change on local production (as in equation (1)) and also the effect of climate change on prices, which in turn depends on how climate change affects production everywhere around the world. As a result, effects on period welfare depend on all of the effects discussed following equation (1), for all regions at once. The value of these effects depends on the marginal value of consumption in welfare.

To derive the effect of climate change on intertemporal welfare in region A, differentiate the maximized value function from equation (A-4), and substitute for p_t from equation (A-3):

$$\frac{\mathrm{d}V_t^A}{\mathrm{d}C} = \frac{\partial W_t^A}{\partial q_{At}^A}\left(1 - \frac{\partial p_t}{\partial Y_t^A} q_{Bt}^A\right)\left(\frac{\partial Y_t^A}{\partial w_t^A}\theta^A + \frac{\partial Y_t^A}{\partial \ell_{Yt}^A}\frac{\mathrm{d}\ell_{Yt}^A}{\mathrm{d}C} + \frac{\partial Y_t^A}{\partial K_t^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C}\right) - \frac{\partial W_t^A}{\partial q_{At}^A}\frac{\partial p_t}{\partial Y_t^B} q_{Bt}^A\left(\frac{\partial Y_t^B}{\partial w_t^B}\theta^B + \frac{\partial Y_t^B}{\partial \ell_{Yt}^B}\frac{\mathrm{d}\ell_{Yt}^B}{\mathrm{d}C} + \frac{\partial Y_t^B}{\partial K_t^B}\frac{\mathrm{d}K_t^B}{\mathrm{d}C}\right) + \frac{1}{1+r}E_t\left[\frac{\mathrm{d}V_{t+1}^A}{\mathrm{d}C}\right].$$

Advancing by one period yields $\mathrm{d}V_{t+1}^A/\,\mathrm{d}C$. We can then substitute $\mathrm{d}V_{t+1}^A/\,\mathrm{d}C$ into the right-hand side. Repeating this process yields:

$$\frac{\mathrm{d}V_t^A}{\mathrm{d}C} = \sum_{s=t}^{\infty}\frac{1}{(1+r)^{s-t}}E_t\left[\frac{\partial W_s^A}{\partial q_{As}^A}\left(1 - \frac{\partial p_s}{\partial Y_s^A} q_{Bs}^A\right)\left(\frac{\partial Y_s^A}{\partial w_s^A}\theta^A + \frac{\partial Y_s^A}{\partial \ell_{Ys}^A}\frac{\mathrm{d}\ell_{Ys}^A}{\mathrm{d}C} + \frac{\partial Y_s^A}{\partial K_s^A}\frac{\mathrm{d}K_s^A}{\mathrm{d}C}\right)\right] - \sum_{s=t}^{\infty}\frac{1}{(1+r)^{s-t}}E_t\left[\frac{\partial W_s^A}{\partial q_{As}^A}\frac{\partial p_s}{\partial Y_s^B} q_{Bs}^A\left(\frac{\partial Y_s^B}{\partial w_s^B}\theta^B + \frac{\partial Y_s^B}{\partial \ell_{Ys}^B}\frac{\mathrm{d}\ell_{Ys}^B}{\mathrm{d}C} + \frac{\partial Y_s^B}{\partial K_s^B}\frac{\mathrm{d}K_s^B}{\mathrm{d}C}\right)\right].$$

Using (A-5), this becomes:

$$\frac{\mathrm{d}V_t^A}{\mathrm{d}C} = \sum_{s=t}^{\infty}\frac{1}{(1+r)^{s-t}}E_t\left[\frac{\partial W_s^A}{\partial q_{As}^A}\left(1 - \frac{\partial p_s}{\partial Y_s^A} q_{Bs}^A\right)\left(\frac{\partial Y_s^A}{\partial w_s^A}\theta^A + (1-\delta)^{s-t}\frac{\partial Y_s^A}{\partial K_s^A}\frac{\mathrm{d}K_t^A}{\mathrm{d}C}\right)\right] - \sum_{s=t}^{\infty}\frac{1}{(1+r)^{s-t}}E_t\left[\frac{\partial W_s^A}{\partial q_{As}^A}\frac{\partial p_s}{\partial Y_s^B} q_{Bs}^A\left(\frac{\partial Y_s^B}{\partial w_s^B}\theta^B + \frac{\partial Y_s^B}{\partial \ell_{Ys}^B}\frac{\mathrm{d}\ell_{Ys}^B}{\mathrm{d}C} + \frac{\partial Y_s^B}{\partial K_s^B}\frac{\mathrm{d}K_s^B}{\mathrm{d}C}\right)\right].$$

The local component to the transitory adaptation channel (i.e., effect of ℓ_{Ys}^{A}) from equation (1) drops out, as current and future allocations to production and investment are optimized for local intertemporal value (i.e., the envelope theorem applies). However, effects on two types of inputs remain. First, climate effects are history dependent. Time t capital stocks are predetermined as of time t, so the effects of climate change on those capital stocks do not drop out of intertemporal value as of time t. Second, other regions optimize their inputs for their own intertemporal value, so effects on inputs elsewhere around the world can have first-order effects on local intertemporal value, via trade.

The first-order change in the welfare of all agents, in both locations and in both the present and future, can be even simpler. If global markets are competitive and fully integrated, then the equilibrium allocation maximizes a global intertemporal welfare function with particular weights on each region and time period. If the global social welfare function has these same weights, then, by the envelope theorem, the adaptation channel from equation (1) in every region of the world drops out of the marginal effect of climate change on global social welfare. However, the capital response does remain, because capital stocks are predetermined as of time t rather than optimized at time t. So past adaptation does affect intertemporal welfare.[2] Moreover, if either the social welfare function uses different weights or global markets have imperfections, then the envelope theorem does not apply and the adaptation channels from equation (1) are part of the effect of climate change on social welfare.

B Papers Included in Trilemma Figure

Adams et al. (1988)

This paper *does not satisfy* desideratum (A) because the analysis is based on calibrating a specific structural model.

This paper *moderately satisfies* desideratum (B). There is adaptation through spatial equilibrium and there is a nuanced representation of hydrologic resource constraints, but there is little representation of agents responding to past or future climate change in ways that would alter yield responses from crop simulation models.

This paper *partially satisfies* desideratum (C). Nothing is estimated directly, but yield responses are based on simulation models based on agronomic understanding of crop growth.

[2] One might wonder why optimization in earlier periods does not mean that marginal changes in the time t capital stock have no effect on time t intertemporal welfare. The reason is that, by standard dynamic analysis, the shadow value of a state variable such as capital obeys a costate equation that does not generally set it to zero.

Mendelsohn, Nordhaus and Shaw (1994)

This paper *largely satisfies* desideratum (A) because the empirical analysis is based on an envelope condition which, although relying on optimizing behavior, is not predicated on a single, specific structural model.

This paper *moderately satisfies* desideratum (B). It uses long-run average temperature as the primary explanatory variable and thereby plausibly captures the persistent nature of climate change. It goes some way towards capturing the anticipated nature of climate change (because anticipation is wrapped up in long-run weather), but anticipation may be very different when applied to the process of future climate change. And it does not capture the widespread nature of climate change because it treats units and their climates as independent of each other.

The paper *does not satisfy* desideratum (C). A primary criticism leveled against cross-sectional analyses like the one in this paper is that they are vulnerable to omitted variable bias. Indeed, Schlenker, Hanemann and Fisher (2005) demonstrate omitted variables bias in the context of this paper.

Schlenker and Roberts (2009)

This paper *fully satisfies* desideratum (A) because the authors leverage the reduced-form empirical approach to implement multiple semiparametric analyses, aiming to achieve robustness to a broader set of potential underlying models.

This paper *does not satisfy* desideratum (B). It uses short-run (growing season) variation in local weather that does not capture persistent, anticipated, or widespread aspects of climate change.

The paper *largely or completely satisfies* desideratum (C). It uses variation in a location's growing season weather across years in order to find variation in weather that is relevant to agricultural production but also quasi-randomly assigned.

Carleton et al. (2022)

This paper *fully satisfies* desideratum (A) because the authors leverage the reduced-form empirical approach to implement multiple semiparametric analyses, aiming to achieve robustness to a broader set of potential underlying models. The analysis is based on a sufficient statistics approach.

This paper *partially satisfies* desideratum (B). The main variation is short-run, local weather that does not capture persistent, anticipated, or widespread aspects of climate change. The analysis includes interactions with longer-run weather, but this interaction only influences outcomes by modifying the estimated effect of short-run weather.

This paper *moderately satisfies* desideratum (C). It uses variation in a location's annual weather across years to identify the effect of abnormal weather on mortality. However, the cross-sectional interactions are not causally identified.

Cruz and Rossi-Hansberg (2024)

This paper *does not satisfy* desideratum (A). The estimates are based on a specific structural model of the economy.

This paper *largely satisfies* desideratum (B). The model includes the effects of spillovers in climate damages, capturing aspects of the widespread nature of climate change. The model rules out some effects of persistent climate change due to the way it models economic behavior.

This paper *partially satisfies* desideratum (C). It regresses productivities and amenities on temperature in a panel environment. However, productivities and amenities are themselves residuals that depend on a particular model, and trade and migration costs are set to make the model perfectly fit observed trade and migration data, so that their identification is ultimately from the model's structure.

Bilal and Känzig (2024)

This paper *largely satisfies* desideratum (A). It estimates a reduced-form relationship between global temperature variation and global production using frontier time series econometrics. It does not do a perfect job because the estimated model is linear rather than semiparametric, in part due to data limitations.

This paper *moderately satisfies* desideratum (B). The use of global variation in principle could capture the widespread effects of climate change, although there is little data to identify this effect. But if successful, the filtration procedure means that the estimates do not capture the effect of anticipated or persistent climate change. The paper includes estimates of dynamics in economic outcomes (output and capital) but in response to global weather shocks rather than permanent climate change.

This paper *partially satisfies* desideratum (C). It is the mirror image of Mendelsohn, Nordhaus and Shaw (1994) in that it relies purely on time series variation whereas the earlier paper relies purely on cross-sectional variation. In principle, this time-series approach might provide as good or better identification than a cross-sectional approach, but in practice it is hampered by limited data (short time series) and retains potential for omitted variables bias.

C An Alternative Formalization for the Adaptation Cost Recovery of Carleton et al. (2022)

Let $C_i(t)$ be the climate in location i at time t, $A(C_i(t))$ be an action that can be used for adaptation at time t, $c(A)$ be adaptation costs at time t, and $y(w_i(t), A(C_i(t)))$ be an outcome at time t, in terms of value. Weather $w_i(t)$ has mean $C_i(t)$ and a stochastic component that is independently and identically distributed over time. Let all functions be continuous and differentiable, ruling out the types of extensive margin adaptation studied by Guo and Costello (2013).

Agents choose time t actions with knowledge of $C_i(t)$ but before $w_i(t)$ is realized. They choose actions to maximize expected net benefits:

$$\max_{A(\cdot)} \int_{t_0}^{\infty} e^{-rt} E\bigg[y(w_i(t), A(C_i(t))) - c(A(C_i(t)))\bigg] \; \mathrm{d}t. \tag{A-11}$$

where the discount rate is $r > 0$. Because there are no intertemporal linkages (for instance, adaptation is assumed completely transitory), this integral can be maximized pointwise, so that optimal actions in time t satisfy:

$$\max_{A(C_i(t))} E\bigg[y(w_i(t), A(C_i(t))) - c(A(C_i(t)))\bigg]. \tag{A-12}$$

Consider climate change from time t_0 to time T. Carleton et al. (2022) are interested in adaptation costs incurred at time T: $c(A^*(C_i(T))) - c(A^*(C_i(t_0)))$, where a star indicates an optimized outcome. By the second fundamental theorem of calculus,[3]

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) = \int_{t_0}^{T} c'(A^*(C_i(s)))\, A^{*\prime}(C_i(s))\, C_i'(s) \; \mathrm{d}s.$$

From (A-12), optimal adaptation in each instant t solves the first-order condition:

$$E\left[\frac{\partial y(w_i(t), A^*(C_i(t)))}{\partial A}\right] = c'(A^*(C_i(t))).$$

[3]We could avoid confusion about the time dimension of climate change by instead expressing this difference as:

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) = \int_{C_i(t_0)}^{C_i(T)} c'(A^*(C))\, A^{*\prime}(C) \; \mathrm{d}C.$$

We maintain the form in the text in order to end up at the equation used in Carleton et al. (2022).

Substituting in pointwise, adaptation costs are

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) = \int_{t_0}^{T} E\left[\frac{\partial y(w_i(s), A^*(C_i(s)))}{\partial A}\right] A^{*\prime}(C_i(s))\, C_i'(s)\ \mathrm{d}s.$$

Move the derivative of A^* inside the expectation operator:

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) = \int_{t_0}^{T} E\left[\frac{\partial y(w_i(s), A^*(C_i(s)))}{\partial A} A^{*\prime}(C_i(s))\right] C_i'(s)\ \mathrm{d}s. \quad \text{(A-13)}$$

To recover the integrand in (A-13), recall regression (5):

$$y_{it} = \beta^{panel} w_{it} + \gamma^{cs,panel} \bar{w}_i w_{it} + \delta_i + \nu_t + \varepsilon_{it}.$$

Carleton et al. (2022) assume

$$E\left[\frac{\partial\, y(w_i, A(C_i))}{\partial A} A'(C_i)\right] = E\left[\frac{\partial y_{it}}{\partial \bar{w}_i}\right], \quad \text{(A-14)}$$

so that, for purposes of adaptation, cross-sectional variation in $\bar{w}_i$ is equivalent to variation in C_i over time. This assumption suits their timeless setting in which the history and future of climate change are irrelevant.

Substituting from regression (5) into (A-14),

$$E\left[\frac{\partial\, y(w_i, A(C_i))}{\partial A} A'(C_i)\right] = \hat{\gamma}^{cs,panel} E[w_{it}].$$

Using that equivalence, adaptation costs in (A-13) become

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) = \int_{t_0}^{T} \hat{\gamma}^{cs,panel} E[w_{is}]\, C_i'(s)\ \mathrm{d}s.$$

Using $E[w_{is}] = C_i(s)$, this becomes:

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) = \int_{t_0}^{T} \hat{\gamma}^{cs,panel} C_i(s)\, C_i'(s)\ \mathrm{d}s,$$

which we can discretize as

$$c(A^*(C_i(T))) - c(A^*(C_i(t_0))) \approx \hat{\gamma}^{cs,panel} \sum_{t=t_0+1}^{T} C_i(t) \Big[C_i(t) - C_i(t-1)\Big].$$

Within this economic environment, a regression that shows how the marginal effects of weather vary cross-sectionally with climate thereby yields how the total costs of adaptation accrue over time as a location's climate changes.

The key assumption is the timeless nature of the decision-making environment. It has two implications. First, optimizing agents equate the flow of marginal costs to the flow of marginal benefits at every instant, so that instantaneous marginal costs can be substituted pointwise for instantaneous marginal benefits (equation (A-12)). Second, adaptation to different climates over time is equivalent to observed cross-sectional adaptation to different climates over space (equation (A-14)). If adaptation were instead not completely transitory, optimization would not permit problem (A-11) to be solved via (A-12) and there is little reason to expect (A-14) to hold.

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